IX.

ON THE LONGITUDE OF WALTHAM, MASS.

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At the request of the Mechanical Superintendent of the American Watch Company's factory in Waltham, Mass., the longitude of their private observatory has been determined as nearly as may be from one night's exchange of signals; and, as that observatory is near a Coast-Survey station, the result has sufficient value to be placed on record for any investigation relative to the problem of station error.

The manner of determining the longitude was as follows: In order to eliminate, as far as possible, the errors in the resulting longitude arising from the lack of a simultaneous action among all the armatures of the electro-magnets used in transmitting and recording the clock-beats at the two stations, it was arranged that both observers should use the same clock, and should, as far as possible, have the same manner of connections at both observatories. The mean-time Bond clock, used for time signals at 97 Water Street, was therefore employed. . This clock, working through the private wires operated by the time-service of the observatory, and by the American Watch Company, recorded its beats simultaneously at Waltham and at Cambridge. At both these stations, local circuits included a chronograph, a relay worked by the Boston clock, and the observing key of the observer.

The observations were made at Cambridge with the broken-telescope transit, by M. Herbst of Poulkova. This is the instrument ordinarily used for time observations, and may be found figured and described in vol. viii. of the "Annals of Harvard College Observatory." It has a clear aperture of 2.75 inches, and a focal length of 32.68 inches, nearly. The pivots are of steel, 1.195 inches in diameter, and sensibly equal. They rest upon V-shaped gun-metal bearings, which are $0^{in}.16$ in breadth, and whose centres are distant

from each other 19.22 inches. The horizontal axis consists of two reversed cones, which terminate in a cube whose single edge is 4.43 inches. The instrument is provided with three positive eye-pieces, magnifying, respectively, 36, 67, and 99 diameters; this last being the one employed in the longitude observations. The reticule of the instrument consists of a series of twenty-five lines ruled by Prof. W. A. Rogers. They are arranged in tallies of five lines each; the equatorial interval between the lines being $3^s.543$, and between the middle lines of the separate tallies $21^s.258$. This form of reticule is found extremely convenient in practice; since a circumpolar star may be observed over the five lines of a single tally, and the instrument can be easily reversed in time to observe a symmetrical tally with the circle opposite its first position. The equal spacing of the lines possible, when they are ruled on glass, reduces by one step the computation of the results. And, by a slight movement of the eye-piece, it is possible in this reticule to obtain a spurious image of a line remarkable for its blackness, without sensibly disturbing the focal image of the star. This is especially convenient in observing faint stars, where it becomes necessary to reduce the illumination so much that the line would ordinarily be invisible. On the other hand, the loss of light by reflection is objectionable; and the writer has not been able to use lines on glass with bright-line illumination for faint stars.

The chronograph by Bond & Sons, situated in the computing room of the observatory, was used.

At Waltham, the observations were made with a transit instrument by Alvan Clark & Sons. It is of the ordinary pattern, has a clear aperture of 2.55 inches, and a focal length of 38.0 inches nearly.

The pivots, 0.972 inches in diameter, rest upon journal bearings 1.00 inch in length. These bearings are distant from each other 17.25 inches. The telescope is provided with a diagonal eye-piece magnifying 64 diameters, and it is reversed by means of a reversing carriage.

The chronograph used is also from the shop of Alvan Clark & Sons; it is of the cylinder pattern, and is controlled by the conical pendulum governor so successfully used by the Messrs. Clark in this connection.

For portable instruments, the formula expressing the relation between the apparent place of a star and its observed place, as affected by errors of observation, may be written

$$\alpha = T + \tau + \Delta\tau + \delta T\,(T - T_0) + A\,a\ (\text{or } A'\,a') + B\,b + C\,c$$
$$- 0^s.021 \cos\varphi \sec\delta,$$

where

α is the adopted right ascension of the star.

T is the observed time of transit.

τ is an approximate correction to the time-piece employed.

$\Delta\tau$ is the correction to τ.

δT is the hourly rate of the time-piece.

$T - T_0$ is the interval in hours between the time of observation and the mean of the times of observation of all the stars combined in one group.

$A\,a$ is the correction for error of azimuth, when the Circle is East.

$A'a'$ is the correction for error of azimuth, when the Circle is West.

$B\,b$ is the correction for error of level.

$C\,c$ is the correction for error of collimation, positive when the Circle is East.

$0^s.021 \cos\varphi \sec\delta$ is the correction for diurnal aberration.

Where the hourly rate is extremely small, and the error of the time-piece is determined at the same instant at both stations, we may write

$$\alpha - (T + B\,b + \tau - 0^s.021 \cos\varphi \sec\delta) = \Delta\tau + A\,a\ (\text{or } A'\,a') + C\,c\,;$$

and, putting for the first member the known term γ, the equation becomes

$$0 = -\gamma + \Delta\tau + A\,a\ (\text{or } A'\,a') + C\,c$$

for each star observed.

In the case of a fixed instrument, we should have

$$0 = -\gamma + \Delta\tau + A\,a + C\,c.$$

The normal equations for the first case are, —

$$\begin{aligned}
0 &= -\Sigma\gamma + \Sigma\Delta\tau + \Sigma A\,a + \Sigma A'\,a' + \Sigma C\,c,\\
0 &= -\Sigma A\,\gamma + \Sigma A\,\Delta\tau + \Sigma A^2\,a + \Sigma A\,A'a' + \Sigma A\,C\,c,\\
0 &= -\Sigma A'\,\gamma + \Sigma A'\,\Delta\tau + \Sigma A'\,A\,a + \Sigma A'^2\,a' + \Sigma A'\,C\,c,\\
0 &= -\Sigma C\gamma + \Sigma C\,\Delta\tau + \Sigma C\,A\,a + \Sigma C A'a' + \Sigma C^2\,c;
\end{aligned}$$

and for the second case,

$$\begin{aligned}
0 &= -\Sigma\gamma + \Sigma\Delta\tau + \Sigma A\,a + \Sigma C\,c,\\
0 &= -\Sigma A\,\gamma + \Sigma A\,\Delta\tau + \Sigma A^2\,a + \Sigma A\,C\,c,\\
0 &= -\Sigma C\gamma + \Sigma C\,\Delta\tau + \Sigma C A\,a + \Sigma C^2\,c.
\end{aligned}$$

Adopting the following values of τ, and using the star places given in the catalogue of 529 stars issued by the Astronomischen Gesellschaft, we have the data, —

For Harvard College Observatory, $\tau = - 16^s.260$;
For Waltham, Mass., $\tau = - 42.855$;

1877, October, $17^d.4$.

Star.	Adopted R. A.	Observed Transit. Harvard College. $b = + 0^s.054$.	Observed Transit. Waltham. $b = - 0^s.048$.
	h. m. s.	h. m. s.	h. m. s.
γ Sagittæ	19 53 19.71	....	6 7 53.39
θ Aquilæ	20 5 0.52	6 19 4.48	6 19 32.50
o Cygni	20 9 47.17	6 23 50.22	....
κ Cephei	20 12 57.25	6 27 59.84	6 27 24.41
γ Cygni	20 17 50.87	6 31 52.66	6 32 20.18
ε Delphini	20 27 38.37	6 41 23.36	6 41 51.12
73 Draconis	20 32 65.58	6 47 4.74	6 47 30.83
ε Aquarii	20 41 4.43	6 55 3.04	....
76 Draconis	20 51 18.86	7 2 18.62	7 5 43.73
ξ Cygni	21 0 29.67	7 14 25.56	7 14 52.87
77 Draco	21 7 54.89	7 21 50.80	7 22 16.62
α Cephei.	21 15 40.17	7 29 33.64	7 30 1.09
β Aquarii	21 25 8.55	....	7 39 28.15
74 Cygni	21 31 63.78	....	7 46 22.05

and the following values of γ: —

Star.	δ	Harvard College Observatory.		Waltham, Mass.	
		Position of Circle.	γ	Position of Circle.	γ
γ Sagittæ	+ 19°.2	....	....	West	— 0s.26
θ Aquilæ	— 1 .2	East	+ 0s.93	West	— 0 .47
o Cygni	+ 46 .4	East	+ 1 .02	....	....
κ Cephei	+ 77 .3	East	+ 0 .68	West	+ 3 .21
γ Cygni	+ 39 .9	East	+ 0 .96	West	+ 0 .13
ε Delphini	+ 10 .9	East	+ 0 .89	West	— 0 .22
73 Draconis	+ 74 .5	East	+ 1 .00	West	+ 0 .98
ε Aquarii	— 9 .9	West	+ 0 .37	....	....
76 Draconis	+ 82 .1	West	— 2 .23	East	— 0 .50
ξ Cygni	+ 43 .4	West	— 0 .04	East	— 0 .70
77 Draco	+ 77 .6	West	— 1 .49	East	— 0 .24
α Cephei	+ 62 .1	West	— 0 .23	East	— 0 .86
β Aquarii	— 6 .1	....	....	East	— 1 .18
74 Cygni	+ 39 .9	....	....	East	— 0 .93

For the stars observed at Harvard College, we have the following equations, where the coefficients are computed with the value of the latitude: —

$$\varphi = + 42^\circ\ 22'.8.$$

For Circle East, —

$$\begin{aligned}
0 &= -0.93 + \varDelta\tau + 0.69\,a + 0.00\,a' + 1.00\,c,\\
0 &= -1.02 + \varDelta\tau - 0.10\,a + 1.45\,c,\\
0 &= -0.68 + \varDelta\tau - 2.61\,a + 4.56\,c,\\
0 &= -0.96 + \varDelta\tau + 0.06\,a + 1.31\,c,\\
0 &= -0.89 + \varDelta\tau + 0.55\,a + 1.00\,c,\\
0 &= -1.00 + \varDelta\tau - 1.99\,a + 3.74\,c;
\end{aligned}$$

and for Circle West, —

$$\begin{aligned}
0 &= -0.37 + \varDelta\tau + 0.00\,a + 0.81\,a' - 1.02\,c,\\
0 &= +2.23 + \varDelta\tau - 4.64\,a' - 7.26\,c,\\
0 &= +0.04 + \varDelta\tau - 0.02\,a' - 1.38\,c,\\
0 &= +1.49 + \varDelta\tau - 2.70\,a' - 4.67\,c,\\
0 &= +0.23 + \varDelta\tau - 0.71\,a' - 2.13\,c.
\end{aligned}$$

A discussion of these equations has shown that the introduction of the term $A'\,a'$ gives values of a and a' sensibly the same.

We may therefore dispense with this term, and our normal equations become

$$\begin{aligned}
0 &= -0.86 + 11.00\,\varDelta\tau - 10.66\,a - 3.40\,c,\\
0 &= -12.06 - 10.66\,\varDelta\tau + 41.53\,a + 40.74\,c,\\
0 &= -34.69 - 3.40\,\varDelta\tau + 40.74\,a + 122.60\,c;
\end{aligned}$$

which give

$$\begin{aligned}
\varDelta\tau &= +0.395,\\
a &= +0.156,\\
c &= +0.242.
\end{aligned}$$

But it is better to substitute the values of $a\,a'$ and c in the equations for the time stars, and thus determine a value for $\varDelta\tau$. We have then, for Circle East, —

for θ Aquarii, the equation $0 = -0.93 + \varDelta\tau + .107 + .242,$
„ o Cygni, „ $0 = -1.02 + \varDelta\tau + .016 + .350,$
„ γ Cygni, „ $0 = -0.96 + \varDelta\tau + .009 + .317,$
„ ε Delphini, „ $0 = -0.89 + \varDelta\tau + .085 + .242;$

and for Circle West, —

for ε Aquarii, the equation $0 = -0.37 + \Delta\tau + .126 - .246,$
" ξ Cygni, " $0 = +0.04 + \Delta\tau - .003 - .333,$ [rej]
" α Cephei, " $0 = +0.23 + \Delta\tau - .110 - .515.$

From which we derive

$$\Delta t = +0^s.562 \pm 0^s.026,$$

and the resulting clock correction

$$\begin{aligned} \tau + \Delta\tau &= -16^s.260 + 0^s.562 \pm 0.026, \\ &= -15^s.698 \pm 0^s.026; \end{aligned}$$

whence the Bond clock is fast of Harvard College Observatory, sidereal time, —

$$15^s.698 \pm 0^s.026,$$

as determined by L. Waldo.

The character of the mounting of the transit instrument at Waltham, combined with a previous discussion of the observations, renders the introduction of the term $A' a'$ superfluous. And in the absence of a determination of the latitude, the latitude is assumed to be

$$\varphi = 42^\circ\ 23'.$$

From the observed stars, we derive the following equations: —

$$\begin{aligned} \text{for Circle West, } 0 &= +0^s.26 + \Delta\tau + 0.42\,a + 1.06\,c, \\ 0 &= +0.47 + \Delta\tau + 0.69\,a + 1.00\,c, \\ 0 &= -3.21 + \Delta\tau - 2.61\,a + 4.56\,c, \\ 0 &= -0.13 + \Delta\tau + 0.06\,a + 1.31\,c, \\ 0 &= +0.22 + \Delta\tau + 0.55\,a + 1.02\,c, \\ 0 &= -0.98 + \Delta\tau - 1.99\,a + 3.74\,c; \end{aligned}$$

$$\begin{aligned} \text{and for Circle East, } 0 &= +0.50 + \Delta\tau - 4.64\,a - 7.26\,c, \\ 0 &= +0.70 + \Delta\tau - 0.02\,a - 1.38\,c, \\ 0 &= +0.24 + \Delta\tau - 2.70\,a - 4.67\,c, \\ 0 &= +0.86 + \Delta\tau - 0.71\,a - 2.13\,c, \\ 0 &= +1.18 + \Delta\tau + 0.76\,a - 1.01\,c, \\ 0 &= +0.93 + \Delta\tau + 0.05\,a - 1.30\,c; \end{aligned}$$

from which we derive the normals: —

$$0 = 0.00 + 12.00\,\Delta\tau + 10.14\,a + 5.06\,c,$$
$$0 = + 8.22 - 10.14\,\Delta\tau + 41.62\,a + 18.07\,c,$$
$$0 = - 37.56 - 5.06\,\Delta\tau + 18.07\,a + 122.88\,c.$$

The solution of which gives

$$\Delta\tau = - 0.175,$$
$$a = - 0.395,$$
$$c = + 0.357, \text{ for Circle East.}$$

But, deriving our final value of $\Delta\tau$ from the time stars only, we have, —

for Circle East, $0 = + 0^s.26 + \Delta\tau - 0^s.17 + 0^s.38,$
$0 = + 0\,.47 + \Delta\tau - 0\,.27 + 0\,.36,$
$0 = - 0\,.13 + \Delta\tau - 0\,.02 + 0\,.47,$
$0 = + 0\,.22 + \Delta\tau - 0\,.22 + 0\,.36;$

and for Circle West, $0 = + 0\,.70 + \Delta\tau + 0\,.01 - 0\,.49,$
$0 = + 0\,.86 + \Delta\tau + 0\,.28 - 0\,.76,$
$0 = + 1\,.18 + \Delta\tau - 0\,.30 - 0\,.36,$
$0 = + 0\,.93 + \Delta\tau - 0\,.02 - 0\,.46.$

From which we derive

$$\Delta\tau = - 0^s.402 \pm 0^s.031,$$
$$\text{and } \tau + \Delta\tau = - 42.855 - 0^s.402 \pm 0^s.031;$$

whence the Bond clock is fast of the sidereal time at the Waltham Station

$$43^s.257 \pm 0^s.031,$$

as determined by C. V. Woerd.

From observations made of the stars ξ Lacertæ, δ Cephei, and ι Cephei, at Waltham, on the evening of Oct. 22, 1877, it was found that on that evening L. Waldo observed an equatorial star $0^s.269$ before C. V. Woerd observed the same star.

The Russian transit pier, at Harvard College Observatory, is 44.5 feet West of the centre of the dome, to which longitudes are usually referred.

If now we correct the observed error of the clock at Harvard College Observatory by the difference of the personal equations of the observers given above, and reduce the position to the centre of the large dome, we have for the assumed error of the clock, as determined at Harvard College Observatory, —

$$+ 15^s.698 \pm 0.026 + 0^s.269 - 0^s.039,$$

which is

$$+ 15^s.928 \pm 0^s.026;$$

and for Waltham we have

$$+ 43.257 \pm 0.031.$$

And their difference gives

$$- 27^s.329 \pm 0^s.040.$$

The most recent determination of the longitude of the centre of the dome of the Harvard College Observatory determines it to be east of Washington

$$0^h\ 23^m\ 41^s.11;$$

whence our final result is that the transit pier in the private observatory erected on Crescent Street, by the American Watch Company, at Waltham, Mass., is East of the centre of the dome of the United States Naval Observatory at Washington, D. C.

$$0^h\ 23^m\ 13^s.78 \pm 0^s.04.$$

HARVARD COLLEGE OBSERVATORY, November, 1877.